The Evolution of Economic Networks

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Abstract

The objective in this paper is to implement a parsimonious agent-based computational model of economic networks whereby agents make strategic decisions based upon profits and information generated through their immediate social network. In this model firms are represented by nodes and the links between each pair of them are the result of a mutually advantageous economic decision. Therefore, links are two-sided or undirected. The economic decision is based on two elements, namely: a myopic profit motive and local information channeled through collaborating firms. Here I endogenize the formation and deletion of links. And also the number of firms (nodes) in the network at each time by allowing firms (nodes) to enter and exit the market. Centrality measures are reported together with firms' profits. The evolution of the network yields higher connectivity and profits when the (positive) externality is high and the rule to exit the market more strict. The higher the network connectivity, the higher the overall profits of firms.

JEL Codes: L22, O33

Keywords: Economic Networks, Agent-based Computational Models

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“A town or city lies at the centre of a number of interlocking catchment areas: there is the *circle* from which it obtains supplies; the *circle* in which its currency, weights and measures are used; the *circle* from which its craftsmen and new bourgeois come; the *circle* of credit (the widest one); the *circle* of its sales and the *circle* of its purchases; and the successive *circles* through which news reaching or leaving the town travels. Like the merchant’s shop or warehouse, the town occupies an economic area assigned by its situation, its wealth and long-term context.” [emphasis added] Fernand Braudel 1979:188.

1. Introduction

If we replace the word ‘network’ by the word ‘circle’ in the quote above we would realize that those networks evolved out of the initiative of a small group of entrepreneurs. Then, others followed those leaders. By this fashion the reach of the networks was gradually expanding out during the early industrialization period. The picture that is captured by Braudel's words shows probably more the result; or a snapshot at a moment of time; of that process.

The way in which these networks overlapped at each moment of time was not the object of choice of those entrepreneurs. As a matter of fact, each network configuration that could have contributed to the development of societies was not necessarily taken into account in the original plans that motivated those forerunners. Yet, it was due to the particularities of each network that a city during the first wave of modern industrialization got access to innovations and discoveries. If we think of networks and their relationship to economic development, we would probably have a clearer way of understading how the `invisible hand' metaphorically used by Adam Smith was actually working.

Networks seen from this perspective can represent how the coordination of economic activities was carried out through different geographical locations (Orsborn and Klein 2007). Here I would propose that these entrepreneurial networks may be seen as `coordination structures' that added value to different economic activities. The particular configuration of these networks at a moment of time can be considered as an unintended consequence of the competitive production process. The general objective of this paper is to understand the evolution of these networks.

I propose an agent-based economic model of formation and evolution of networks whereby agents make strategic decisions based on economic variables and information generated through their immediate social network. It is important to bear in mind that the particularities of a network in a static snapshot may not be an equilibrium situation but rather one of disequilibrium. Then, a simulated environment will help to appreciate this better than other conventional tools.

In the next section, I review the literature that motivated this study. Section 3 presents the research questions. Section 4 introduces concepts that will be used in the rest of the paper. Section 5 presents my strategy in modeling this evolving network. Section 6 shows the reference simulations of the model. Section 7 reports the results of experiments after manipulating key parameters in the model. The last section summarizes the main findings so far.

2. **Literature Review**

Two strands of literature one theoretical and the other empirical dealing with social networks yield obvious, though different conclusions concerning the evolution of cooperation within a society of individuals that interact based upon strategic and self-interested behavior. The more abstract and game-theoretical models of social networks sustain that in coordination games in social networks multiple equilibria do emerge as stochastically stable states. This is in contrast to previous results, such as Young (1998), as I will explain in the subsequent section. In line with this, also the study of multiplayer prisoners' dilemma games leads to the results that cooperation emerges on a sparse matrix rather than on close-knit networks. The second more empirical strand is based upon studies of actual economic sectors yielding as results that particularly in high dense networks (not sparse) underlies the productivity and economic growth of certain localized industries.

Before I describe the details of these two strands of literature a few definitions are in order. I will follow more closely a graph theoretical approach in doing this (see Beineke and R. J. Wilson (1997). A *network* is a set of nodes wherein any pair of nodes is connected, at least, by a link. A *fixed network* is a set with a given and finite number of nodes, and a fixed configuration of links among the nodes. A *dynamic (endogenous) network* is a set with a given and finite number of nodes, but a variable configuration of links among the nodes.

This variable configuration is usually the result of an endogenous formation process for links. An *evolving network* refers to a variable (even stochastic) number of nodes and link configurations among them (Romero 2006, Cowan et. al. 2006).

2.1 Game-theoretical Approach to Social Networks

Jackson and Watts (2002) study fixed and endogenously formed networks whereby players are playing coordination games with their neighbors. Each player only interacts with those other players whom are directly linked to it, and each link is formed after mutual consent. Also, there are costs of forming links. This results in games with only two pure Nash equilibria where the payoffs matrix is specified in such a way to model the Pareto equilibrium also as the risk-dominant strategy.

In the fixed network case they analyze three variations: a lattice or complete graph, a circle graph, and a star shaped graph. Stochasticity is added when agents choose their strategies. Here their main contribution is their result for a star shaped network that is in stark contrast to the conventional result; e.g. Young (1998). The latter claims that for any fixed network players always converge to the risk-dominant strategy. On the contrary, Jackson and Watts claim that the two equilibria may be chosen, thus all players may be playing the risk-dominant equilibrium or playing the efficient but not risk-dominant equilibrium in other periods.

In the case in which agents choose not just their strategies, but also whom to play with, their main result is that there is multiple equilibria and players may coordinate even in those equilibria that are neither risk-dominant nor efficient. By manipulating the cost structure of the game, they even go farther to claim that even for fixed networks they may exist multiple stochastically stable states. Thus concluding that the conventional results; e.g. the risk-dominant solution as the unique stochastically stable state; are sensitive to the particularities of agents' behaviors and interaction technologies.

Hanaki et. al. (2007) address the emergence of cooperation where individuals' behavior and interaction structures are evolving. In this setting there is a dynamics on the network generated by the rules that govern individual behavior, and also a dynamics of the network that is generated by the rules governing social behavior. The rules of individual behavior are based on each agent playing a prisoners' dilemma game with each of its surrounding or local neighbors. However, each agent can choose either to cooperate or defect with its whole neighborhood; i.e. it cannot play a different strategy against any other agent within its neighborhood. Because this is a simulated environment the population of players are actually playing a multiplayer prisoners' dilemma with a changing subset of other agents that at each period may be part of its neighborhood. Moreover, each player can imitate the most successful strategy-- measured by its payoff-- of the last period by one of its neighbors. Also they can break or create a link with another agent to modify their neighborhood. There is not an exogenous upper limit for the number of neighbors that a given agent can have during the simulation.

Concerning the interaction dynamics this is determined by the marginal increase or decrease in benefits per player from either breaking or creating a link with other player. One important element in this model is the incorporation of triadic relationships through which an agent can find a new partner. Nonetheless, an agent can decide to create a link with an agent randomly drawn from the population at large. There are costs in both cases; that is, for breaking and creating a link. Moreover, there is an additional procedure to decide whether to trust a new partner. Here two different settings are implemented, namely a full and a zero information case about the history of plays by the new partner in previous periods.

Their main result is the following: “cooperation can persist in sparse, dynamic networks of effectively unlimited size, and in fact tends to fare better in large networks than in small ones.” pp.1049. They emphasize how assortative matching of partners reinforces cooperation (as in Tullock 1980). But also how allowing defectors to be selected by highly trusting cooperators expands this cooperation. During the report of their results they also acknowledged that a “higher average proportion of cooperating players does not necessarily mean that the population average payoff is higher.” pp. 1004. This point is relegated to a footnote where they mention that despite this result there still exist a positive correlation (0.48) between the average proportion of cooperating players and the population average payoff. But from the main text one of their main results is that a high-cost regime for agents' interaction is what determines both the sparseness of the network and its greater level of cooperation achieved in relation to when there is a lower cost of interaction.

2.2 Empirical Social Network Analysis

The main particularity of economic networks at the producer level is that they change from period to period. The firms representing the nodes may have changed. The networks of raw material providers/retailers and clients may change from period to period. Nonetheless, there is a core or nucleus of clients to whom the seller frequently sells and a core of providers from whom usually it buys. These are their permanent clients and providers. But those not permanents clients and providers can be called casual ones. This is network complementarity between embedded and `arm's length ties'.

Castilla (2003) and Castilla et. al. (2000) focus on a static network where only embeddedness is studied and thus highlighted as the main driving force of the creation of capital in Sillicon Valley. Next, I explain two examples that were elaborated by Castilla et. al. (2000) and Uzzi (1999). The first work is about Silicon Valley and how the development of that region is generated through the networks of venture capitalists, educators, engineers, lawyers, trade groups, and so on. Regarding the conformation of technological firms a special focus is given to employees and referees, managerial, and information networks that are generated and transmitted through different links or channels among firms. The second one is about bank-borrowers networks in the Chicago area.

Castilla et. al. call attention to the fact that: “Extensive labor mobility creates rapidly shifting and permeable firm and institutional boundaries and dense personal networks across the technical and professional population. The ability of Silicon Valley to restructure itself when conditions change through rapid and frequent reshuffling of organizational and institutional boundaries and members (…“recombinant” process) is one of the factors that underlie the dominance of Silicon Valley…” pp. 220. Their analysis show how the creation of capital in Silicon Valley is benefited and fostered by the positive externalities created due to the high degree of density and the openly competitive environment among different networks related to a given firm. An intense competition and high mobility of resources allows for a fast rate of learning of adaptation to the new conditions of the market. One important characteristic that they pointed out is the fact that much of the know-how or informal knowledge produced by this interaction among technological firms remains local.

Using techniques of social network analysis with data collected by journalists they are able to trace--since 1947 up to 1986-- the evolution of the network of firms, managers, educators and others. This was what contributed to the beginnings of projects as Intel and the like. Those individuals or firms with a high degree of centrality (connected to a lot of others) and those that play the role of `crucial linkage' to reach others are discovered. Hence, entrepreneurial spirit, willingness to support innovative ideas, but specially networks externalities are the key elements identified by them lying at the great development of Silicon Valley.

A visual representation is in the network from which initial public offers (IPOs) (data from 1999) are originated in Silicon Valley. Figure 1 shows three different kinds of organizations that interact and collaborate to give birth to a new enterprise. These are: investments banks, law firms, and accounting firms. Furthermore, the issuing firm is not portrayed. There is a link between any two firms (from the same or different industry) whenever both are involved in the same IPO. The length of the line also conveys relevant information, namely it is inversely proportional to the number of co-participations. Thus is a proxy for the strength of the link. The more co-participations, the stronger the relationship (i.e. the shorter the link).

The main result is that a particular kind of network; defined by centrality and degree of connectivity; determines particular outcomes. That is, different types of relationships that may exist among the actors of any network. In a posterior work by Castilla (2003), he compares the degree of connectivity or density of the network of venture capital firms in Sillicon Valley to the one in Route 128 (Massachusetts). He found that the higher number of projects and amounts of money invested in California are a consequence of the higher connectivity among firms through different industrial sectors and within each of them.

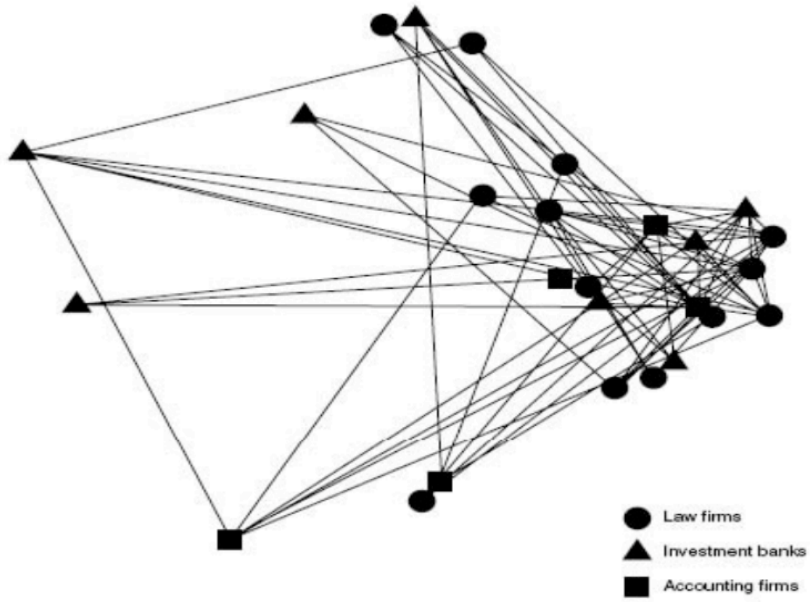

Figure 11.5. Network of IPO deals in the information retrieval services industry in California.

Figure 1

On the other hand, Uzzi (1999) carried out an analysis of the effects of social embeddedness of networks in corporate financial dealings. An important contribution of this paper is the triangulation between social network analysis, statistics, and original data collected through field research. The sample included 2400 small or medium size companies and eleven medium size (less than 500 employees) banks in the Chicago area. His focus is upon the credit networks or the bank-borrower links and their effects on the amount and cost of loans obtained. The first pair of hypotheses is: a) if bonds or social attachments created (and the longer this relation exists) among managers and bankers increase the probability of getting a loan; and b) if given this, the cost will be lower. Data from the fieldwork pointed out that bankers and managers do care about how to establish a social relationship with one another beyond the cold numbers. Because to get to know each other gives them information that is not easily found in figures and increases the degree of trust in their relationships.

The other pair of subsidiary hypotheses tested by Uzzi is: the likelihood to get financing increases if a firm has access to a mix of embedded and arm's length ties. In other words, if a mix of bonding and bridging social relationships in different networks is important to arbitrage opportunities and reduce search costs. The other hypothesis, then, is if costs of financing are lower when a firm has access to these two kinds of social networks. Another way to put this is that if a firm only has been focused on cultivating only one kind of these networks' links (bonding or bridging) it will be less successful getting loans and reducing the costs per loan.

An important concept explored by Uzzi is related to this mix of bonding and bridging networks what he referred as to `network complementarity.' In his own words: "Networks high in complementarity produce premium outcomes because the features of different ties reinforce one another's advantages while mitigating their disadvantages." pp. 491.

The econometric tests yielded these results: the social network bonding links did not affect the probability of a given firm to get loans, but it does affect the price or interest rate of the loan. The latter is in agreement with field data. In regards to the tests about network complementarity these pointed out that these kinds of combined network links do produce optimal benefits relative to networks only of one type or another for a firm.

3. Where All This Lead Us? Research Questions

To what extent the game-theoretical results of network games (Goyal et. al. 2007) explain the empirical evidence of actual social networks in the market? What it is reported in field research on social networks may be just one type of equilibrium explained by the models. But here my purpose is toward building a rationale of: how social networks contribute to the development of commercial ties? In a more general vein: How do firms coordinate to produce technology through networks; i.e. economic networks?

An economic or entrepreneurial network is formed by a profit motive and also social links. In this model nodes represent firms and the link between each pair of them is the result of a mutually advantageous economic decision. The environment is an industrial sector where firms interact locally but contribute to a global evolving network of technological innovation (Cowan et. al. 2006). Links in this case are not one-sided or directed but two-sided or undirected.

Moreover, the temporal dimension of the process will be studied by how long it takes to the network of firms to evolve a network structure (topology). This will serve us to answer the following questions: how do the model's parameters and rules of interaction affect the network evolution? Under what networks profits may be greater? Are certain network topologies more prone to generate coordination among firms? and, in general: Can this be a part of that intangible capital that accounts for endogenous economic growth through knowledge?

4. The Environment

The agents are firms that will interact within an industry. A firm may cooperate or not with another firm. There will be direct relationships that will be established pairwise, and indirect ones that are a consequence of the former kind of relationships. That is to say, each firm only focus on the relationships that establishes directly with other firms. This pairing of firms can be understood as a contract to collaborate whether in the funding of a new enterprise with innovative ideas or contributing with knowledge to a particular investment project. This keeps some similarity to what happens in places such as the Silicon Valley, but I keep the model rather general. There is neither a market demand nor a production technology. I have relied on Wasserman and Faust (1994), Scott (1991), and Goyal (2007) to write the next two subsections.

4.1 Definitions

Firms are represented by a set of nodes N = {1,…, n} where $n \geq 2$ and a finite number. Their pairwise relationships are links or edges denoted by $g_{ij} \in \{0, 1\}$ for nodes i and j. Where g_{ij} takes value 0 when the two nodes are not connected and 1 otherwise. Here I will consider only undirected links, which in this context means that both nodes mutually accept to establish and maintain the link. Let G^t be the network formed by a set nodes and its links at a time t. There is a set of networks $\Gamma = \int_0^T G_t \, dt$ representing each of the G networks along time.

A neighborhood of agent i is the set of all its neighbors with whom is directly connected represented by $N_i(g) = \{j \mid g_{ij} = 1\}$. The degree of node i is the number of direct neighbors $d_i(g) = | N_i(g) |$ in a given network G. The first order neighborhood of node i is N_i. The second order is $N_i \cup \{ N_z \mid z \in N_i \}$. Other higher order neighborhoods can be defined in a similar manner. Let $\overline{d}$ be the maximum degree for a given network. The degree distribution of the network is denoted by P, and the frequency of nodes with degree d is P(d).

The following are relevant type of networks. The complete network, g^c, is the one where every node has the same degree and this is equal to n -1. The empty network, g^e, which is not connected or is the degree zero. A connected network is where there is a path between any two nodes even though is not a complete network.

Other important concept is a *walk,* which is a sequence of nodes whereby two nodes are linked. Here a node or a link may be included more than once in the walk. The length of the walk is the number of links it crosses or the number of nodes involved minus one. A *trail* is a walk in which all crossed links are distinct. In turn, a trail in which every node is distinct is a *path.* The length of the path is the number of links that involves. There is a *shortest path* between nodes i and j; called its *geodesic distance* in network G which is measured by its length and denoted by t_{ij}. For every node i in network G there may exist a set of shortest paths to every other node j. Whenever there is no path between any two nodes in G then their geodesic distance is $t_{ij} = \infty$.

4.2 Measuring a Network

When a network G is connected its average distance between nodes or path length is

$$L = \frac{\sum_{i \leq j}^{n} t_{ij}}{n(n - 1)} \qquad (1)$$

This is useful to know how close is an agent (firm) to another one and how easy or fast information or knowledge can be transferred in a network.

The centrality of an agent in a network refers to its prominence; i.e. how relevant or critical is the presence or absence of this agent in the network. This is measured by

$$C_d = \frac{d_i(g)}{n - 1} \tag{2}$$

A related measure is the degree centralization of a network G. If there is a node i^* with the highest degree centrality C_d^* then

$$C_d(g) = \frac{\sum_{i=1}^{n}\left[C_d^{\cdot} - C_d\right]}{n - 2} \tag{3}$$

The density of a network G measures the proportion of potential links present in it. It is expressed as a ratio of actual links to the maximum possible ones. This is

$$\Delta = \frac{\sum_{i}^{n} d_i}{n(n - 1)} \tag{4}$$

Another measure that will be introduced is the clustering coefficient. This captures the overlapped links that exist among the neighbors of agent i or what proportions of its neighbors are also neighbors. This is defined for any node i as

$$C_l = \frac{\sum_{l \in N_i(g)} \sum_{k \in N_i(g)} g_{lk}}{d_i(d_i - 1)} \tag{5}$$

Finally, the total clustering coefficient is the sum of all individual clustering coefficients. That is, $C = \sum_{i=1}^{n} C_l$. I will use these concepts and measures in different sections along the paper.

5. The Evolving Network

At the beginning, independently of any value of the parameters and exit treatments, there will be only one firm in the market. Then, firms make their appearance in the environment one by one per period. ‘New’ firms arrive and propose to form a link or economic relationship with ‘old’ firms. The latter should decide whether to accept such a proposal. Those firms that are unsuccessful; i.e. the ones that held negative profits for several periods of time; leave the industry. Therefore, a dynamic process is recreated in which firms enter and leave the market affecting the economic relationships that have also been formed dynamically.

Every firm contributes to the technological innovations throughout the network. But every firm arrives in the deterministic fashion I explained previously. Thus at this stage the model does not include elements of stochasticity. The particular topology of the overall network changes every time period. Because some firms are entering while others are leaving the market. Firms are also risk neutral.

The flow of innovations in this industry is the result of not just each firm contribution, but more importantly of the connectivity of the network that all firms form. I will draw on Jackson and Wolinsky (1996) formalization of the ‘connections model’ from now on. Let w_j be the market valuation of firm j’s potential contribution to firm i's innovative endeavors[1]. Then, the accumulated value after a period t for firm i (which value w_i is of itself) interacting with every other firm j in the network Gt is

$$V_i(G_t) = w_i + \Sigma_{j \neq i} \partial^{t_{ij}} w_j \tag{6}$$

where δ ∈ (0, 1) is a parameter that represents the transferability factor or how firm i gets access to the innovations of firm j via intermediate links and other firms in the network. This is expressed by $d_i(g_t)$ that is the degree of node i for a network g at t. Thus, the connection with node j is indirect via the local neighborhood of node i. The positive externality deteriorates the farther is firm j from i. There are costs; denoted by c_{ij}; of forming links between any two firms. Therefore, profits for firm i per unit of time are given by

$$\pi_i(G_t) = w_i + \Sigma_{j \neq i} \partial^{d_i(g_t)} w_j - \Sigma_{j \in G} c_{ij} \tag{7}$$

The dynamics of the network is given here at two levels. Firstly, as I mentioned before there is not a fixed number of firms during the simulated time. As a consequence links between firms cannot be fixed either. Both, the number of firms and their links are permitted to evolve during the simulation. By doing this, the state variables of the firms are also altered every period during the experiments.

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[1] 'Innovations' within this literature have also been interpreted as 'knowledge' within each firm. A link is formed whenever this 'knowledge' is purposefully shared or diffused throughout the network; e.g. Cowan and Jonard (2006). I am avoiding this usage since I consider knowledge a more abstract category of thought than information, for instance. See Polanyi (1974: 69 -260) and Hayek (1937, and 1945) for further distinctions about knowledge and the relevance of its tacitness. So in my case innovation is the same as 'new' information.

When a new firm arrives to this industry it proposes to form a link with an incumbent firm. When there are more than two firms the incumbent firm is randomly chosen from the new firm's neighborhood. Next, I define a myopic pairwise dynamics in which:

> i) A new link is created as long as both firms do not get worse off by establishing this relationship, and at least one of them is strictly better off (Jackson and Watts, 2002).
>
> ii) An old link is severed if at least one of the firms who formed it exits the market due to accumulated negative profits for m successive periods. Otherwise, it is maintained.

The first point is standard in the study of endogenous network formation when there is a finite set of nodes during a simulation. While the links appear and disappear from the network at each period yielding certain network topologies. However, I should emphasize that in this model; in contrast to previous approaches; nodes also appear and disappear from the environment (or rather interface). The number of nodes and links are endogenous at each period. This is the main contribution of this paper.

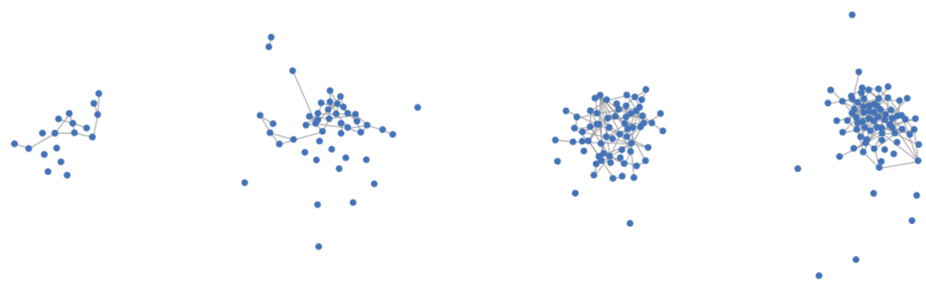
Before to proceed I should, also, point out that in this model I incorporate the notion of *ex ante* and *ex post* gains for the firms forming or removing links. Thus, I assumed that at the beginning of each period firms get to know the market valuation of others, which are represented by the w_is. But the realized profits are given by (7) at the end of each period. This is also a different approach from the one implemented in Carayol and Roux (2005) and König(2008) in which case ‘knowledge’ is represented by ‘innovations’ that arrive every period according to a known probabilistic distribution. Leaving unspecified the distinction I introduce here regarding ex-ante versus ex-post profits.

The second point, on the other hand, it is the result of merging two processes. The first one is a firm exiting the market due to consecutive negative profits (e.g. four quarters), and the second one is the deletion of the link(s) or economic relationship(s) between that firm and their directly connected firm partners (i.e. neighboring nodes). Here lies another innovation of the paper whereby the evaporation of nodes is paired with the deletion of their direct links.

6. Simulation Results

Figure 1 depicts how firms and their linkages evolve through time for a typical run. Periods should be assumed larger than a day. It could be months and even quarters. The particular length of the period will only make sense when an empirical validation against actual data is carried out. The externality parameter (δ) is set at 0.95. I, also, here only present an exit rule (treatment) for firms leaving the market. *This rule states that a given firm with negative profits or* *unconnected from any other firm for more than 4 consecutive periods will exit the market*.

Below, I show snapshots of the evolution of the network at different time periods.



a) t= 40 Ave degree = 1.18	b) t = 200 Ave degree = 2.89	c) t = 500 Ave degree = 3.18	d) t= 1000 Ave degree = 3.2

Figure 2. Evolution of Firms Network. Degree mean values across firms.

At the bottom of each panel in Figure 2 the average degree is reported. Note how this measure increases between panel a) and b) and from then to c). Between c) and d) is more or less the same. I won't make strong statistical claims at this moment (see more in section below). I just wanted to point out the monotonic increase of the average degree of the network.

In figures from 3 through 6 the evolution of some key aggregate variables is presented. Figure 3 depicts two variables namely the number of 'surviving' firms and the 'surviving' links. It should be reminded that in this model firms enter the market at a constant rate of 1 per unit of time. Plus the *exit rule* previously mentioned implies that several firms may exit the market by the end of each period. Note that the number of firms and links increase more or less *pari passu* up to 200 periods. After that firms grow at a decreasing rate whereas links grow increasingly faster until they start to fluctuate after 700 periods around a value of 140.

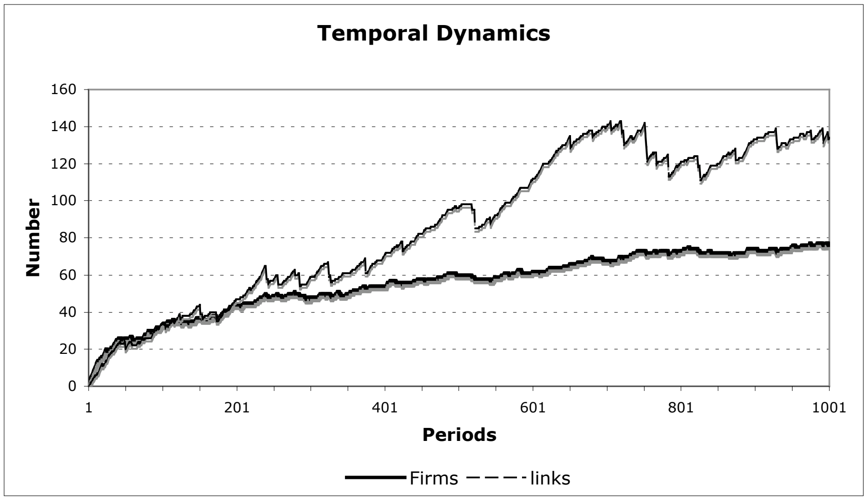


Figure 3.

Figure 4 shows the evolution of average profits and average degree of the network of firms at each time step. It is not surprise that the correlation between both variables is 0.86. This stems from the profit equation per firm as specified in (7). This, of course, does not mean that any firm with the highest degree due to its higher number of related firms will always have the highest profit. Because in the profit equation (7) every firm also faces costs per each related firm that it keeps. In addition these costs vary from period to period. And there are no fixed costs, all costs are variable.

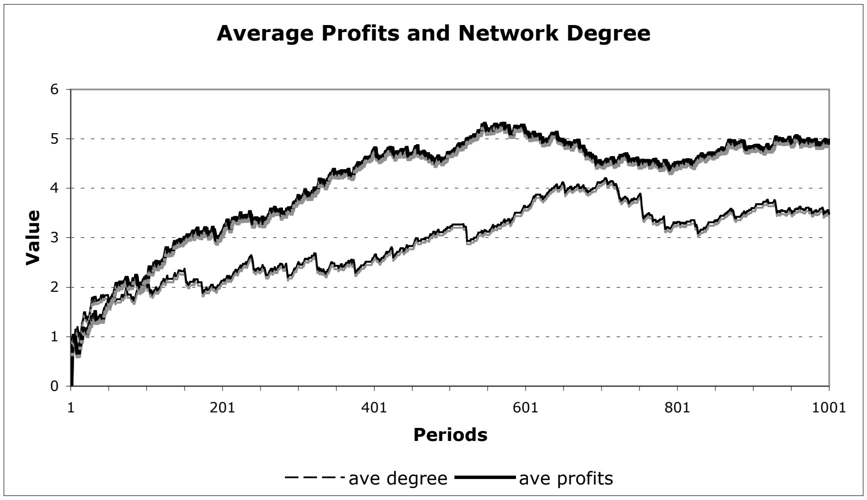


Figure 4.

Figure 5 presents the average fraction of firms in the *giant component* along the simulation. Note that after 200 periods this fraction reaches 80% of the whole population of ‘surviving’ firms. This population at any time step may include the firms belonging to the giant component, firms that are part of other component(s), and temporarily unconnected firms. This fraction fluctuates between 80% and 90% after 300 periods.

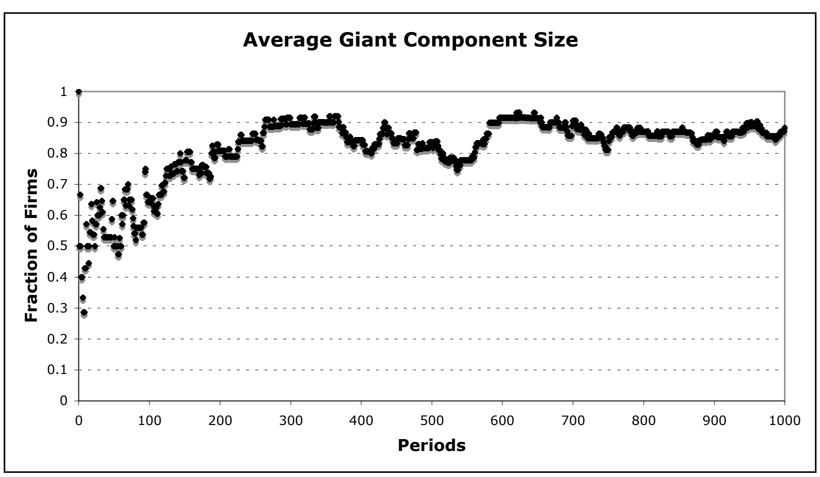


Figure 5.

Figure 6 presents the degree distribution of the firms network after 50 simulations each one measured at t =1000. This result shows a power law relationship when the distribution is measured in logs. The power coefficient is close to 1 and statistically significant at 95% level of confidence. I will point out that here this power law relationship is found in spite of the absence of a 'preferential attachment' mechanism during the network formation. Next section presents a more formal analysis of robustness of the results.

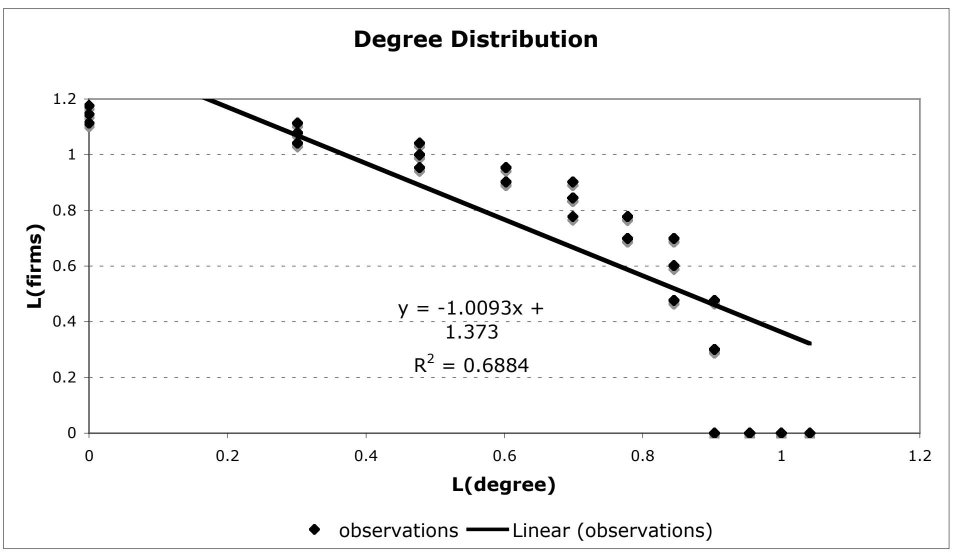


Figure 6. Fifty simulations each one at t=1000.

7. Experiments

What is the effect of the externality parameter (δ) on the network degree and firms' profits? Since average degree and profits are positively correlated the effect of increasing (decreasing) the externality parameter should be the same for both. Recall that the externality parameter is measuring how fast the rate of knowledge is spread over the network. The higher it is the faster knowledge is being transferred throughout. I run experiments for δ = {0.05, 0.5, 0.95} and two exit rules. A firm will leave the market if for more than 4 or 12 consecutive periods has been having negative profits or has been unconnected from others; i.e. its degree is zero. This adds up to six experiments.

Figure 7. δ = {0.05, 0.5, 0.95}. 50 runs per experiment each one for t = 1000.

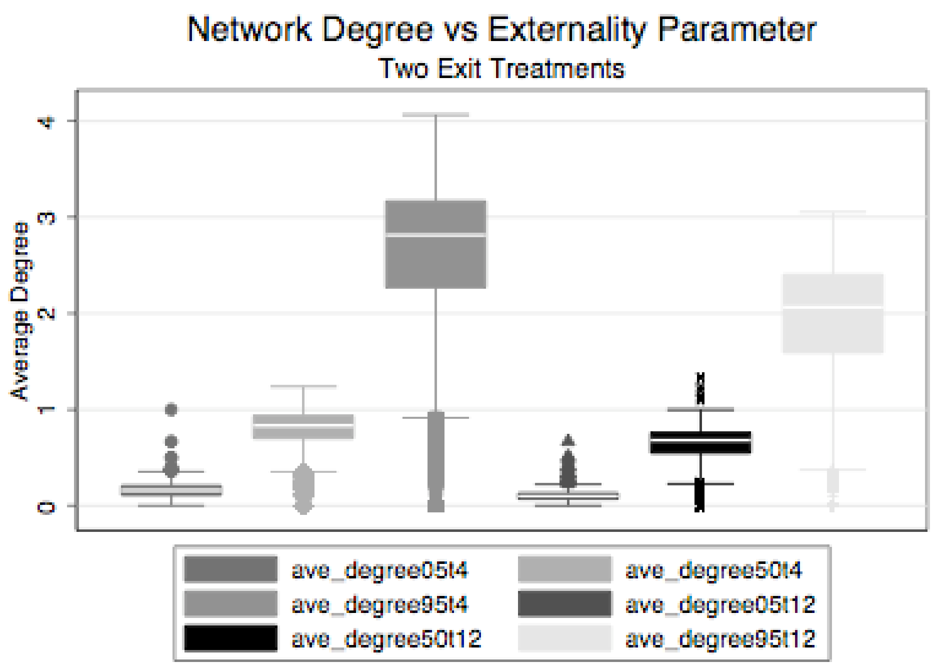


Figure 7 reports the average degree of firms after 50 runs. The results of the six experiments (3 δ values times 2 exit rules) are showed from left to right. The externality parameter takes the values of {0.05, 0.5, 0.95} per each exit treatment. Exit treatments are also read from left to right. The first exit treatment (I) refers to the same rule applied to the results showed in section 6. While the second exit treatment (II) just increases that value to 12 consecutive periods but it works in the same fashion as mentioned in the previous paragraph. Thus, in the figure I report the ordered pair values of {δ, exit rule} = {(0.05, I); (0.5, I); (0.95, I); (0.05, II); (0.5, II); (0.95, II)}. Note that within each exit treatment the average network degree monotonically increases with the value of the externality parameter as expected. Then, I proceed to test if the means across exit treatments per each value of δ are statistically equal or not with an unpaired difference means test with unequal variances. That is, I compare the means difference between the pairs (0.05, I) and (0.05, II); and so on (I implement two-sided and one-sided tests). So that there will be three means difference tests. The results reject the null hypothesis of equal means (i.e. a zero value for the difference of means) at the 5% level of significance. For instance, it can be claimed that the average network degree when δ = 0.95 and the firms are interacting under exit rule (I) is statistically different (and higher) than the average network degree when δ = 0.95 but firms are interacting under exit rule (II).

Next, Figure 8 reports the values of average profits after 50 runs again. The number of total experiments is the same as for the average network degree case. Also, the ordered pair values {δ, exit rule} are the same as before. It is also observed that within each exit treatment average profits monotonically increase with the value of the externality parameter. A means difference test was also applied to determine whether these values are statistically different across exit treatments exactly as I did for the average network degree case. The results of the three means difference test ordered as before yielded also a rejection of the null hypothesis in each case. Again, I can claim that the average profits when δ =

0.95 and the firms are interacting under exit rule (I) is statistically different (and higher) than the average profits when δ = 0.95 but firms are interacting under exit rule (II). The higher the network degree or connectivity, and the most competitive the market, the higher the overall profits of firms.

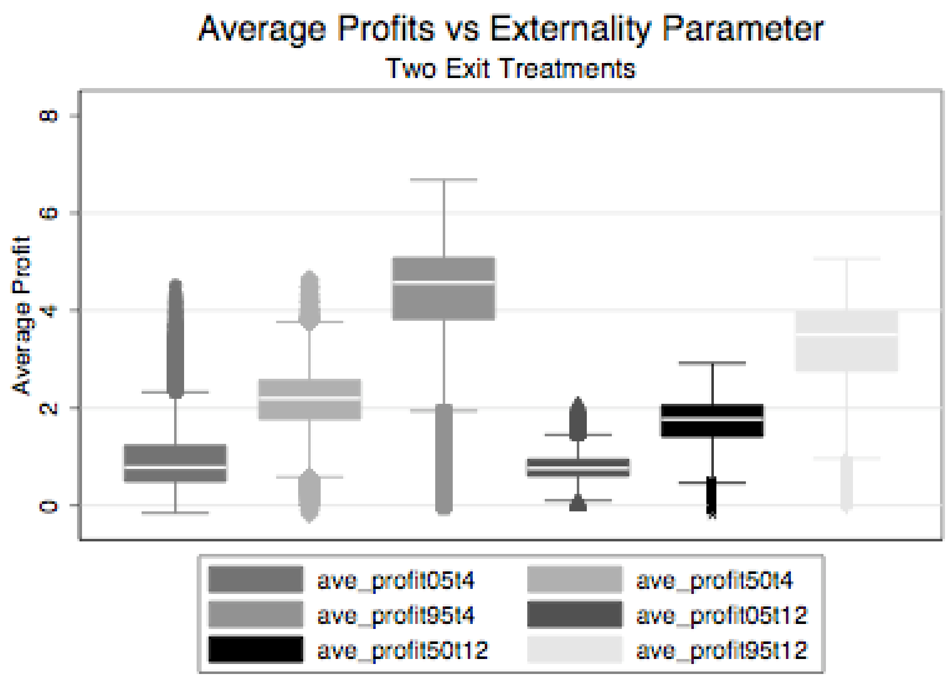


Figure 8. δ = {0.05, 0.5, 0.95}. 50 runs per experiment each with t = 1000.

8. Concluding Remarks

Castilla (2003: 125) found that the average degree of the overall network of venture capitalists in Silicon Valley is 2.8 while the same network statistic for Route 128 (Massachusetts) is 1.5. This is one of the findings on which he based his conclusion that the greater frequency of cooperation in Silicon Valley is what explains its greater economic success. The empirical average degree reported there is pretty much the same reported here in Figure 7, i.e. 2.78 for the first exit treatment and $\delta = 0.95$. Whereas the second exit treatment and the same externality value yield an average degree of 2.01.

My goal here is not to make a precise quantitative calibration of the model. Rather at this stage a qualitative calibration is what I have in mind (Axtell and Epstein 1994). But, even as it stands the model may shed light on the empirical differences between Silicon Valley and Route 128 venture capitalists' networks, for instance. As a matter of fact, here was also found that the higher the network connectivity the greater the profits; or the economic efficiency loosely defined (Romero 2006). The model, of course, cannot yet portrait the trade-off or complementarity between bonding and bridging links reported by Uzzi (1999). Nonetheless, the model at this stage is more able to explain a type of regional development due to factors within a hub like the one in Silicon Valley.

Goyal (2007: 20-4) summarizes the features of empirical networks across several domains. Including corporate web site, coauthors, sexual contacts and R&D networks. He concludes: "[social and economic networks] have low average degree relative to the total number of nodes, the distribution of degrees is unequal, clustering is high, and the average distance between nodes is small." (pp. 23-4) The model presented here, also yields a low average degree relative to nodes, see Figure 3. An unequal distribution of degrees was reported in Figure 6. I have reported here neither the clustering coefficient nor the average path length but I would not be surprised if, in fact, it mimics the general pattern reported by Goyal.

These results were yielded by the model and closely match the ones from empirical networks. Yet I did not follow in building the model more traditional approaches: such as preferential attachment mechanisms, random networks, or small world networks. I based my model more on simple economic grounds and local information. Thus, providing a more credible microeconomic behavior of the agents.

Thomas and Griffin (1996) and Lin and Shaw (1998) present complementary works on *supply chain networks* and how coordination through top management techniques have become less and less of practical use in multi-national process of production. Knowledge and practices are so much distributed throughout the supply chain network that no one can manage it only relying on global information. In general, these supply chain networks are comprised by: raw material providers, manufacturers, assemblers, warehouses, and retailers. In turn, these networks can be subdivided into three types of categories, namely: buyer-vendor networks, production-distribution networks, and inventory-distribution networks. The model presented here falls into a production network category but it can be extended to include distribution and the demand aspects of an artificial industrial environment.

Industries ranging from auto, computer hardware, airlines' services, is where most of the case studies are found. The general point in all of them is that coordination in these industries that show vast 'economies of scope' have recently tended to spread their production processes as moving from vertically integrated towards more flatter networks. This has resulted from the search for coping with uncertainty and adaptation to a more competitive environment. By doing so, leaders in these industries have been able to discover opportunities not known or existent before. This kind of coordinative processes that go beyond a particular firm or even region can account for an important part of economic growth not included in more traditional models.

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