DIVIDE AND CONQUER: NOISY COMMUNICATION IN NETWORKS, POWER, AND WEALTH DISTRIBUTION

(Preliminary draft, May 16, 2003)

Wilson Perez

Department of Economics, Cornell University

Abstract

The political and economic evolution of countries such as Nigeria and Congo challenge the conclusions of Mancur Olson's (1993, 2000) "stationary bandit" model. In this paper I argue for the necessity of an alternative model where the balance of political power and the distribution of income are endogenous. Such a model is developed in this paper as a game played between a ruler who has to decide the distribution of income and a group of agents/citizens who can communicate through a network. The citizens also have the opportunity to revolt if they are not happy with the distribution, but if too few of them revolt, they are defeated and get zero consumption. On the other hand, a successful revolt increases the consumption level of the rebels while the ruler gets nothing. Communication through the network is noisy, however, which could preclude the emergence of common knowledge and collective action among the citizens. The ruler could take advantage of this to make an unfair income distribution. The formalization of the game is accomplished using such concepts as p-beliefs and p-dominant strategy (Monderer and Samet ,1989, and Morris and Shin, 2002). We use this model to offer some reflections about how rent-seeking governments manage to survive.

I.- INTRODUCTION

In Mancur Olson's (1993, 2000) model of the origin of the state, an itinerant bandit decides to settle down, seizes and holds a territory, and becomes a respected ruler of its inhabitants. Since the ruler maximizes his income, which is the product of the tax rate and the tax base, he has to take into account the incentive-distorting effect of taxation. So, although in this model the ruler holds total political power, he does not want to expropriate his subordinates completely so he will set a low tax rate that leaves them an adequate incentive to produce. Moreover, it is in his interest to provide public goods, such as enforcing property rights and private contracts among his subordinates, and providing them peace and order, because that will enable his subjects to increase their production—i.e., to enhance the tax base. In the same way, building roads and bridges could be a profitable investment for the ruler. In the end, the subjects enjoy a better and safer environment and a higher income than they would in an anarchic environment, at the mercy of itinerant rovers.

We can see that there are, at least two implicit assumptions in Olson's argument. The first is that current production technology does not allow the ruler to get as much labor as he wants from his subordinates by coercion alone. Otherwise, his subjects would be slaves instead of entrepreneurs/citizens. The second assumption is that the main (or only) source of the ruler's income is taxation of citizens' production. This ensures that he will be interested in encouraging private production. However, there are historical and current examples of countries where at least one of these assumptions fails. Let's focus on the second one.

His subjects' production is not always the ruler's main source of income. There are many countries where the citizens, because of the lack of capital, technology, and a safe environment for business, are able to produce very little. Nevertheless, some of these countries generate relatively large incomes from exporting raw materials such as oil, diamonds, and gold, which are mainly appropriated by powerful elites. This description fits the economic situation of various African countries, and in section IV we apply our model (developed in section II) to two of them: Nigeria and Congo. In these cases, the "ruler" does not find it profitable to invest in making his subordinates more productive, but instead prefers to appropriate the country's rents as much as he can. In other words, in the place of a production/taxation model, we see a distribution/appropriation model.

In such a scenario, a ruler holding total political power—as in Olson's model—will appropriate 100% of the export income. This is not realistic, of course, since the citizens will claim their share of the country's wealth and, being the majority, could overthrow the ruler. In other words, there is a political limit to the ruler's extraction, originating in the citizens' ability to react to the regime's actions. Thus, in order to understand the political economy of these poor countries with rich subsoil and powerful elites, we need a model where both the balance of political power and the distribution of income are endogenous. To understand this, we must answer the following questions: Why do rent-seeking governments survive? If no regime could survive a generalized revolt, why do people not simply get rid of such governments? What defines how much political power a government has over its citizens? How does this power affect wealth extraction and distribution?

Looking for an explanation of rent-seeking governments' survival we could use, for example, Olson's (1965) "free rider" argument. We would argue that the overthrowing of such government is a public and non-excludable good, which would be enjoyed both by people who did revolt and by people who did not. Hence, it would be rational for an individual to avoid the cost of arising against an extractive regime (especially a repressive one) by letting other people overthrow it. Since every rational subject will draw the same conclusion, the rent-seeking administration will survive. This approach to collective action (or inaction) has been widely analyzed (see Oliver 1993 and the extensive bibliography there) and we don't wish to add to that debate. Instead, we offer a different explanation of this phenomenon based on the lack of common knowledge of the distribution of wealth.

To approach these issues, we develop a model with a ruler and *n* citizens, who have a utility function *U(.)* that depends only on the consumption of a unique kind of good available in this economy. The citizens are the nodes of a communication network. The structure of the network is exogenous and could be understood as the result of geographic restrictions (such as natural barriers among villages or natural links such as navigable rivers) or cultural conditions (different languages, castes, or social classes, regionalism, social norms of inclusion and exclusion). People can send information through the network, but this communication could be defective in the following sense: Let's assume that the consumption level of an agent has two possible states, *M* and *Z*; a person linked to that agent will observe the true state with probability *a* and the other with probability *1-a*. The information about that agent's consumption will travel through the network, suffering the possibility of such deformation at every link. We'll call *1-a* the "noise level" and *a* the "the channel capacity". The reasons for such distortion could be cultural (such as norms against "flaunting one's wealth" and lack of trust among citizens[1] or among ethnic groups) and represent a simplification of the noise present in every communication process.[2]

The total amount available of the consumption good is exogenous. By this assumption, in this model there will be no room for an incentive-distorting effect of the ruler's extraction, so we can focus on the influence of the political factor on the extraction level. It's the ruler's job to distribute the total amount of consumption good, which is equal to $(n+1)M$ for some $M>0$, among the citizens and himself. To do so, the ruler and the citizens play a one-shot two-stage allocation game. In the first stage the government

[1] Originating, for example, in the presence of secret police among the citizens.
[2] See Shannon 1948.

has to allocate one of the two consumption levels (Z or M, $0<Z<M$) to each citizen. The leftover will be the ruler's consumption.

The ruler, of course, would like to assign Z to as many citizens as possible, but after the allocation is made it is the citizens' move. Essentially, every agent has to decide, privately and simultaneously, whether to revolt against the ruler or not. If a given agent decides not to rebel, his consumption level would be whatever is assigned by the regime. If he attacks the ruler and at least $\hat{f}-1$ other agents also decide to revolt, they will defeat the ruler and realize an individual consumption level of $\hat{M}$, where $Z<\hat{M}<M$, while the ruler will get zero consumption. If fewer than $\hat{f}$ agents revolt, the regime will prevail and maintain its original consumption while each rebel will get zero utility. That means we are assuming that the goods the defeated rebels would have consumed will be thrown away; the government therefore will not get any benefit from a defeated uprising.[3] On the other hand, assuming that only uprising agents will benefit from a triumphant revolution, we put aside the "free rider" effect and focus only on the consequences of the absence of common knowledge of the distribution of wealth, as explained below.

In principle, then, if at least $\hat{f}$ agents receive a consumption level of Z, it is in their interest to revolt. In fact, if this happens in a perfect information environment, there will be two Nash Equilibria in the second stage: one where the underprivileged agents rebel and one where nobody does. In this paper, we focus on the analysis of the former case, because it is difficult to imagine how a ruler could rely on the latter when making its decisions. Since the agents get information about one another through a noisy

[3] This is equivalent to assuming that when the government is able to defeat an uprising (i.e. when fewer than $\hat{f}$ agents revolt), it has to expend Z to defeat a rebel.

communication network, however, no agent knows the payoff of any other agent with certainty. This complicates the emergence of collective action but does not make an uprising impossible. Therefore the ruler, when allocating the good, should take into account the probability of being defeated if he assigns Z to at least $\hat{f}$ citizens.

We use the *p*-believe theory developed by Monderet and Samet (1989) and Morris and Shin (2002) to understand when a successful revolt is possible and how this possibility conditions the ruler's maximum expected utility. We find that this maximum depends on the channel capacity and, more interestingly, on the degree of network connectivity. In particular, the lower the channel capacity and the less connected the network, the higher the expected utility of the ruler and, in some interesting cases, the higher the number of deprived citizens. In addition, we find that it could be in the government's interest to provide M to some citizens, in particular those who have relatively more connections. We use this model first to reflect on how rent-seeking governments survive and then to shed light on the political economy of countries such as Congo and Nigeria

The paper is organized as follows: Section II explains the model; Section III present two examples, and Section IV applies the model and discusses the results.

II. THE MODEL.

In this section we introduce some notation and formalize the game. We then explain how it works in an incomplete-asymmetric information environment. Finally, we analyze how coalitions against the ruler could emerge and how such possibility shapes the ruler's best response.

The basic game.

We consider a set of $n+1$ agents: agent 0, the ruler, and the citizenry $N = \{1,...,n\}$. Each agent has an increasing and concave utility function, $U(x)$, which depends only on his consumption $x\,(x \geq 0)$. We normalize the utility function such that $U(0) = 0$.

Among these individuals there are bilateral and symmetric relationships called communication links. We note *ij*, the link between i and *j*. Let's call $\Gamma^N = \{ij \,/\, i \in N,\, j \in N\}$ the set of all possible links among the agents in *N* and Γ a set of links (i.e. $\Gamma \subseteq \Gamma^N$). A communication network is a non-oriented graph (N, Γ) where the players are the nodes, connected by the bilateral links in Γ. The shortest path between two agents i and j is called the geodesic,[4] and the number of links along such a path is called the degree of separation between i and j, noted as d_{ij}. We consider only networks (N, Γ) that interconnect every pair of agents in *N*. The network structure is common knowledge among the agents.

Now we define a one-shot, two-stage "allocation game." In the first stage the ruler distributes the total amount of consumption good, exogenously set as $(n+1)M$, assigning some non-negative amount X_i of the good to each citizen i, where $X_i \in Q = \{M, Z\}$, $0<Z<M$. Hence the set of strategies for agent zero is $S_0 = \{\{X_i\}_{i=1,2,...,n}, X_i \in Q\}$. Note that in principle the ruler would be able to allocate *M* to every citizen and still get *M* for himself.

[4] If there is more than one distinct geodesic from i to *j*, we choose one randomly for driving the information from i to *j* and vice versa. So, we always talk about "the" geodesic connecting two agents. The agents will rely only on the information coming through a geodesic.

The second stage is the citizens' turn to play. Each one of them has two strategies available: $S_i = \{A, C\}$ for $i = 1,2,...,n$, where C stands for "accept X_i and do not fight the ruler" and A stands for "do not accept X_i and attack the ruler." The outcomes will depend, however, not only on the individual's decisions but also on the other citizens' decisions. In particular, it is necessary that a coalition of at least $\hat{f} > 1$ individuals fight the ruler to defeat him. If such a coalition arises, the ruler will in fact be defeated and get a consumption level of zero. Then every member of the triumphant partnership will get a new allowance of $\hat{M}$, where $M > \hat{M} > X$, and hence the utility for every member of such a winning group will be $U(\hat{M})$. Meanwhile the citizens who didn't participate in the revolt (either triumphant or defeated) won't be affected, keeping their utility at $U(X_i)$. We assume $\hat{M}$ is such that $\frac{U(Z)}{U(\hat{M})} > 1/2$.[5] On the other hand, if the attacking coalition has fewer than $\hat{f}$ members, the ruler will prevail and the members of the defeated group will lose their whole endowment, which will be thrown away, and get a utility of 0. Therefore, for the citizens the payoffs are as follows:

$$U_i = \begin{cases} U(X_i)\, if \quad C \\ 0 \quad if \quad A\, and \quad f < \hat{f} \\ U(\hat{M}) \quad if \quad A \quad and\ f \geq \hat{f} \end{cases}$$

Every citizen chooses his strategy privately and simultaneously. Of course, the players maximize their expected utility. The ruler's utility will be $U\left((n+1)M - \sum_{i=1}^{n} X_i\right)$ if not defeated and zero if defeated.

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[5] We'll see that this assumption facilitates the calculations.

We can see that if the endowments $\{X_i, i = 1,2,...,n\}$ are common knowledge, and $f \geq \hat{f}$ citizens get $X_i = Z$, then there is a Nash equilibrium where a coalition arises and defeats the ruler, since it is common knowledge among these *f* citizens that it is in everybody's interest to fight the ruler and get a utility of $U(\hat{M}) > U(Z)$. Hence, in this situation, the best the ruler could do is to assign *M* to $n - \hat{f} + 1$ citizens and *Z* to the other $\hat{f} - 1$ agents. In that way, $n - \hat{f} + 1$ citizens will get a utility $U(M)$ and will not participate in any rebellion, while the remaining agents are getting a utility of $U(Z) < U(M)$, but cannot consolidate a coalition sufficiently strong to defeat the ruler.[6] Hence, agent zero will ensure a utility equal to $U\left(M + (\hat{f} - 1)(M - Z)\right)$. In this perfect-information environment, if at least $\hat{f}$ citizens get $X_i = Z$ there is also a Nash equilibrium where nobody challenges the ruler. Such a Nash equilibrium could allow the ruler even to assign $X_i = Z$ to everybody, but we don't find such a case interesting because, again, it's difficult to imagine how a ruler could rely on such an equilibrium to define his best response.

The incomplete information game.

Let's assume the endowment X_i is known just by *i* itself and by the ruler. To get some knowledge about the others' endowments, every agent has to rely on the communication network. Now assume additionally the communication in such network is noisy, meaning that if agent i has an endowment $X_i = q_l, l \in \{1,2\}$, $q_l \in Q$, agent j (who

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[6] We're assuming that no re-distribution of endowments (in any situation) is possible among any group of agents, which is logical in this non-cooperative environment.

is one link away from i) will receive a signal $\hat{X}_{ji}$, which is a random variable with distribution $a_{hl} = P(\hat{X}_{ji} = q_h \mid X_i = q_l), h = 1,2$. Such probability distribution could be arranged in the markovian transition matrix: $\Pi = \begin{bmatrix} a_{11} & 1-a_{22} \\ 1-a_{11} & a_{22} \end{bmatrix}$. Additionally we note $\overline{X}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ if $X_i = M$ and $\overline{X}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ if $X_i = Z$. Hence, if agent j is one link away from i, the distribution of the signal *j* receives is $\Pi\overline{X}_i$.

The information about i's endowment will travel through the network suffering such possible distortion at every link. For example, consider an agent *r* and suppose the geodesic from i to *r* is $\{ij, jr\}$. This means *j* receives an unclear message, $\hat{X}_{ji}$, about X_i and *r* receives an unclear signal, $\hat{X}_{ri}$, about the information *j* has gotten about X_i. Then the distribution of $\hat{X}_{ri}$ is $\Pi^2\overline{X}_i$. In general, the signal $\hat{X}_{si}$ an agent *s* receives about i's endowment, given the degree of separation between them, d_{si}, has the distribution $\Pi^{d_{si}}\overline{X}_i$. Note the journey of the signal through a geodesic is a Markov chain. We assume signals with different origin are stochastically independent, that is $\{\hat{X}_{ij}\}_{i\in N}$ and $\{\hat{X}_{ik}\}_{i\in N}$ are independent if $j \neq k$.

It will make the calculation easier if we assume $a = a_{11} = a_{22}$ and $a \geq 1/2$. Such matrix has two useful properties: $\prod^k \to \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ when $k \to \infty$, and using the notation $\Pi^k = \begin{bmatrix} a_{11}^{(k)} & a_{12}^{(k)} \\ a_{21}^{(k)} & a_{22}^{(k)} \end{bmatrix}$ we have: $a_{11}^{(k)} > a_{11}^{(k+1)} \geq 1/2, k = 1,2..$. That allows us to define the "noise level"[7] in the communication network as $1-a$. If the noise level is zero there will be common knowledge of the endowments. By the contrary, if noise level is ½, the higher possible in this context, the signals wont carry any useful information (see below). Also, the farther the signal travels, the information it carries is less useful. We assume this communication environment is also common knowledge.

Given the communications structure, the endowments $\{X_i\}_{i=1}^{n}$ define the probability distribution of the signals each agent will receive. To be precise, we define a probability space $\{\Omega, \Re, \hat{P}\}$ where, using the convention $\hat{X}_{ii} = X_i$, we have:

$\Omega = \left\{\omega = \left\{\hat{X}_{ij}\right\}_{i=1,\ldots,n}^{j=1,\ldots,n}; \hat{X}_{ij} \in Q\right\}$ The sigma-algebra $\Re$ is the power set of Ω,

$\Re = \{\Lambda / \Lambda \subset \Omega\}$, and $\hat{P}$ is the probability distribution $\hat{P}\left(\left\{\hat{X}_{ij}\right\}_{i=1,\ldots,n}^{j=1,\ldots,n} \middle| \{X_i\}_{i=1,\ldots,n}; a; (\Gamma, N)\right)$.

Where we have added the parameters ***a*** and (Γ, N) to emphasize the dependence of this probability on the noise level and the network structure, respectively. The exact way to calculate $\hat{P}$ is described in appendix 1. Notice this probability is known only to agent zero, and is different from the citizens' priors, defined as follows.

Let be the incomplete information game $\left\{\Omega, (P_i)_{i=1}^{n}, (\Psi_i)_{i=1}^{n}, (U_i)_{i=1}^{n}\right\}$, where P_i is i's prior probability distribution on omega, defined in the same way of $\hat{P}$, which depends on the endowments distribution $\{X_i\}_{i=1}^{n}$, the channel capacity a and the network structure (Γ, N). But the problem is that citizen i does not know the endowments, except his own,

[7] We also refer to a as channel capacity.

so i has to rely on a prior about the others agents' endowment. Following Morris and Shin (2002) we assume such prior distribution is uniform,[8] that is:

$P_i(X_j = Z) = P_i(X_j = M) = 1/2, \forall j = 1,2..n, j \neq i$.

Ψ_i is player i 's partition of the state space Ω . If $\sigma \subset N \setminus \{i\}$ and

$\Psi_i^\sigma = \left\{ \left\{ \hat{X}_{kj} \right\}_{k=1,\ldots,n}^{j=1,\ldots,n} / \hat{X}_{kj} \in Q, \hat{X}_{ij} = Z \quad if \quad j \in \sigma, \hat{X}_{ij} = M \quad if \quad j \notin \sigma \right\}$ then i 's partition will be: $\Psi_i = \left\{ \Psi_i^\sigma, \sigma \subset N \setminus \{i\} \right\}$. Since agent i sees only the signals he has gotten, $\left\{ \hat{X}_{ij} \right\}_{j=1}^{n}$, two events ω and ω' are in the same set of his partition if they yield him the same collection of signals. We abuse the notation calling $\Psi_i(\omega)$ the set in i 's partition where ω belongs to, hence $\Psi_i(\omega) = \Psi_i(\omega')$ if and only if ω and ω' are in the same set of i 's partition.

Finally, $U_i : S \times \Omega \to R$ is player i 's payoff function, with $S = S_0 \times S_1 \times .. \times S_n$ and the strategies and payoffs as described above.

The emergence of coalitions.

The information the agents receive, therefore, is incomplete (since every citizen gets just an imprecise signal about the other agents endowment) and asymmetric (since every agents knows exactly its own endowment and the signals could be different for each citizen). Thus, every agent can just infer the endowments and signals that his partners could have gotten, and then can just conjecture the strategies they could adopt. So, it's clear that common knowledge about payoffs is lost in this game as long as $a < 1$ and we

[8] Morris and Shin (2002) call this prior "Laplacian", because it follows Laplace's "suggestion that one should apply a uniform prior to unknown events from the principle of insufficient reason." See *op cit* p.5 and 6.

now have to understand how collective action could arise in such environment. As Morris *et al* (1995), page 145, explain:

> When payoffs in a game are not common knowledge, the outcome depends not only on players' beliefs about payoffs, but also on their beliefs about others' beliefs about payoffs, and on their beliefs about others' beliefs about their own beliefs, and so ad infinitum.

To approach this problem we use the concepts of *p*-belief operators created by Monderer and Samet (1989) and *p*-dominance and *p*-dominant equilibrium, developed by Morris and Shin (1995 and 2002). Following Morris and Shin (2002), define $|g|$ as the set of states in the incomplete information game where payoffs are given by *g*:

$$|g| = \left\{ \omega \in \Omega / U_i(s,\omega) = U_i\left(s, \left\{ \hat{X}_{kj} \right\}_{k=1,\dots,n}^{j=1,\dots,n} \right) = U_i\left(s, \left\{ X_i \right\}_{i=1}^{n} \right) = g_i(s), \forall s \in S, i = 1,\dots,n \right\}$$

We say $U_i\left(s, \left\{ \hat{X}_{kj} \right\}_{k=1,\dots,n}^{j=1,\dots,n} \right) = U_i\left(s, \left\{ X_i \right\}_{i=1}^{n} \right)$ because the payoffs will depend only on the strategies and on the endowments. Then define a pure strategy Nash equilibrium s^* of a complete information game, *g*, as *p*-dominant equilibrium if each player's action is a best response[9] whenever he assigns probability at least *p* to his opponents choosing according s^*:

$$\sum_{s_{-i} \in S_{-i}} \lambda(s_{-i}) g_i(s_i^*, s_{-i}) \geq \sum_{s_{-i} \in S_{-i}} \lambda(s_{-i}) g_i(s_i, s_{-i})$$

For all $i = 1,\dots,n;\quad s_i \in S_i$, and for all λ probability distribution on S_{-i} such that

$\lambda(s_{-i}^*) \geq p$.

We need to recall the definition of *p*-belief operators. Let be an event $E \subset \Omega$. The event "i *p*-beliefs E" is noted $B_i^p(E)$ and defined as $B_i^p(E) = \left\{ \omega \in \Omega / P_i\left(E \mid \Psi_i(\omega) \right) > p \right\}$.

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[9] As usual we note $S_{-i} = S_0 \times S_1 \times .. \times S_{i-1} \times S_{i+1} \times .. \times S_n$ and $s_{-i} = \{s_0, s_1, .., s_{i-1}, s_{i+1}, .., s_n\}$

The event “E is p-believed” is $B^p(E)=\bigcap_{j\in N} B_j^p(E)$. Finally, the event E is common p-belief at state ω if it is p-believed that it is p-believed, and so on, up to an arbitrary number of levels. We note the set of such ω as $C^p(E)$. At this point we need:

Lemma 4.2 from Morris and Shin (2002). If s^* is a *p-dominant* equilibrium of the complete information game *g*, then every incomplete information game $\left\{\Omega,(\pi_i)_{i=1}^n,(\Psi_i)_{i=1}^n,(U_i)_{i=1}^n\right\}$ has an equilibrium where s^* is played with probability 1 on the event $C^p(|\, g\,|)$.

Now, we have the instruments to solve our game. We’ll use backward induction in the following way. Let’s assume the ruler has defined $\{X_i\}_{i=1}^n$, where at least $\hat{f}$ citizens got *Z*. The regime’s goal is then to measure the event where a group of at least $\hat{f}$ citizens will play “attack” with probability one. Using that measure, the ruler could calculate the expected utility such allocation $\{X_i\}_{i=1}^n$ will yield to him. Doing this exercise for each possible allocation, the government will be able to choose his best response.

Let’s identify the events that the ruler should count as “attacked by at least $\hat{f}$ agents”. First note in a perfect information game, “attack” for i receiving *Z* and “do not attack” for i receiving *M* is a NE. Also, “attack” will be a best response for i only if $X_i = Z$ and the probability *p* agent i assigns to the event “at least $\hat{f}-1$ other citizens will attack” is p>1/2, since then the expected utility of attacking is $pU(\hat{M})+(1-p)U(0)=pU(\hat{M})\geq U(Z)$, and we did assume $\dfrac{U(Z)}{U(\hat{M})}>1/2$. Second, note although revolts of more than $\hat{f}$ citizens are possible, from the ruler’s point of view it’s enough to account the events where $\hat{f}$ citizens play "attack".[10] From now on, σ will note $\sigma \subset N$ and $\#(\sigma) = \hat{f}$.

Third, note that if we're trying to identify the events where a specific group σ of deprived citizens will arise, we have to care only about events where $\hat{X}_{ij} = Z \quad \forall i, j \in \sigma$. To see this, consider the case where some $s \in \sigma$ receives a signal *M* about $k \in \sigma$, that is $\hat{X}_{sk} = M$. Then, we know: $P\left(X_k = Z \middle| \quad \hat{X}_{sk} = M\right) = a_{21}^{(d_{sk})} < 1/2$. That means the probability agent ***s*** will assign to the event "everybody in σ got *Z*" is less than a half. If that is the case, "attack" will never be a *p*-dominant strategy for *s*, since for that it is necessary *p*>1/2.

Therefore, we can focus on the event $E = \left\{\omega \in \Omega / \hat{X}_{kj} = Z; k, j \in \sigma\right\}$. In this event, every citizen in σ gets *Z* and also receives a signal $\hat{X}_{kj} = Z$ from every member of the group. We want to find the condition for *E* to be common *p*-believed by σ . This is stated in the next lemma.

Lemma 1. The necessary and sufficient condition for $E = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in \sigma\right\}$ being common *p*-believed in *E* (i.e. $C^p(E) = E$) is $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right) \geq p$.

(See the proof in appendix 2).

We're ready to say when a revolt of $\hat{f}$ citizens will happen.

[10] Because if f citizens, $f > \hat{f}$ play "attack" on the event ω , obviously $\hat{f}$ agents will play attack on such event ω , and the results –from the ruler's point of view- will be the same.

Proposition 1. If for some $\sigma \subset N$ with $\hat{f}$ members, where $X_i = Z$ for $i \in \sigma$, it is true that $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right) \geq p$, for some $p \geq \frac{U(Z)}{U(\hat{M})}$ then "attack" will be played with probability 1 for the agents in σ in the event $E = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in \sigma\right\}$.

To prove this, first note if for some $\sigma \subset N$ with $\hat{f}$ members $X_i = Z$ for $i \in \sigma$, then in a complete information game $i \in \sigma$ will play "attack" in one of the two NE. For some $p \geq \frac{U(Z)}{U(\hat{M})}$, that will be also a *p*-dominant equilibrium. Using the assumption that $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right) \geq p$, Lemma 1, and Lemma 4.2 from Morris and Shin (2002), we can conclude "attack" will be played with probability 1 for σ in the event $E = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in F\right\}$. That completes the proof.

We're ready to tell the ruler the events when a coalition of at least $\hat{f}$ members will play "attack", given the endowments $\{X_i\}_{j \in N}$. Such event is:

$$\Phi = \left\{\omega \in \Omega / \exists \sigma \, \forall i \in \sigma \quad : \left\{\hat{X}_{ij} = Z\right\}_{j \in \sigma}, \hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; t, k \in \sigma\right) \geq p, \, p \geq \frac{U(Z)}{U(\hat{M})}\right\}$$

Agent zero is maximizing his expected utility:

$$P.1 \quad \underset{\{X_i\}_{i=1}^n}{Max} \quad \left(1 - \hat{P}\left(\Phi; a, (\Gamma, N), \{X_i\}_{i=1}^n\right)\right) U\left((n+1)M - \sum_{i=1}^{n} X_i\right)$$
$$s.t \; X_i \in Q \quad i = 1,2..n$$

This maximum exists, since the set of possible $\{X_i\}_{i=1}^n$ is finite. Given the two-stage structure of this game and how the citizens define their strategies once the allocation is made, the ruler could use backward induction to find its best strategy.

Now let's note as $U((\Gamma, N), a)$ as the maximum expected utility the ruler could get, given the network and the channel capacity. The next corollary explains that the more (less) connected the network and/or the higher (lower) the channel capacity, the lower (higher) the expected utility will get in this game.

Corollary 1: If $\Gamma_1 \subset \Gamma_2$ and $a_1 < a_2$, then:

$U((\Gamma_2, N), a) \leq U((\Gamma_1, N), a)$ and $U((\Gamma, N), a_2) \leq U((\Gamma, N), a_1)$

Proof: Let's assume $U((\Gamma, N), a_2) > U((\Gamma, N), a_1)$. Call $\{\tilde{X}_i\}_{i=1}^n$ the allocation where the maximum $U((\Gamma, N), a_2)$ is achieved, that is:

$U((\Gamma, N), a_2) = \left(1 - \hat{P}\left(\Phi; a_2, (\Gamma, N), \{\tilde{X}_i\}_{i=1}^n\right)\right) U\left((n+1)M - \sum_{i=1}^n \tilde{X}_i\right)$. If we apply such allocation to the game with lower channel capacity a_1, we will get that the set of events where "attack" will be played by at least $\hat{f}$ citizens is smaller when the channel capacity is lower: $\Phi_1 = \Phi\left(a_1, (\Gamma, N), \{\tilde{X}_i\}_{i=1}^n\right) \subset \Phi\left(a_2, (\Gamma, N), \{\tilde{X}_i\}_{i=1}^n\right) = \Phi_2$, since if $\omega \in \Phi_1$, then $\exists \sigma \subset N$ such that $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma; (\Gamma, N), a_1\right) \geq p$. Hence, for the same group of citizens it is also true that $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma; (\Gamma, N), a_2\right) \geq p$, that is $\omega \in \Phi_2$. Even more:

$1 - \hat{P}(\Phi_1; a_1) > 1 - \hat{P}(\Phi_2; a_2)$, therefore:

$$\left(1 - \hat{P}(\Phi_1; a_1)\right) U\left((n+1)M - \sum_{i=1}^n \tilde{X}_i\right) > \left(1 - \hat{P}(\Phi_2; a_2)\right) U\left((n+1)M - \sum_{i=1}^n \tilde{X}_i\right)$$

Using our initial assumption, that means

$\left(1-\hat{P}(\Phi_1;a_1)\right)U\left((n+1)M-\sum_{i=1}^{n}\tilde{X}_i\right)>U\left((\Gamma,N),a_1\right)$, but this is not possible since the right hand side of the inequality represents the maximum utility level the ruler could get with channel capacity a_1. The proof for different network connectivity is almost the same. That completes the proof.

The next corollary should also be noted:

Corollary 2: For any network structure[11] (Γ,N) and any group σ (with $\hat{f}$ members) receiving an allocation of *Z*, there is a noise level $1-a$ such that "attack" will be played for the agents in σ with probability 1 on the event $E=\left\{\omega\in\Omega/\hat{X}_{ik}=Z;k,i\in\sigma\right\}$. On the other hand, there is always a noise level $1-a$ such that "attack" will never be played for the agents in σ.

Proof: For any fully connected network structure (Γ,N) and any coalition σ of $\hat{f}$ members receiving an allocation of *Z*, $\hat{P}\left(\hat{X}_{tk}=Z\mid X_k=Z;k,t\in\sigma\right)$ is a continuous and increasing function of *a*, with value 1 for *a*=1 and value less or equal than ½ for *a=1/2*. Then we apply proposition 2. That completes the proof.

III.- EXAMPLES.

Example 1

Let's analyze the simplest network: two agents connected by one link. The number of agents needed to defeat the ruler is, of course, two. Hence the set of strategies for agent zero is the set of endowments:

[11] Recall we're assuming the network is fully connected, i.e. there is always a geodesic connecting any couple of agents.

$S_0 = \{\{X_1 = M, X_2 = M\}, \{X_1 = Z, X_2 = M\}, \{X_1 = M, X_2 = Z\}, \{X_1 = Z, X_2 = Z\}\}$

The problem for the ruler is, therefore, to decide whether to assign *Z* to only one agent or to both of them. If only one agent gets *Z* the utility of the ruler is $U(M-Z)$. Let's find the cases when the ruler gets a higher expected utility assigning *Z* to both agents. So, assuming he's doing so we get:

The probability $\hat{P}\left(\{\hat{X}_{12}, \hat{X}_{21}\}; a, (\Gamma, N), \{X_1, X_2\}\right)$ is:

$\hat{P}\left(\hat{X}_{12} = M, \hat{X}_{21} = M\right) = (1-a)^2 \leq 1/4$

$\hat{P}\left(\hat{X}_{12} = M, \hat{X}_{21} = Z\right) = (1-a)a$

$\hat{P}\left(\hat{X}_{12} = X, \hat{X}_{21} = M\right) = (1-a)a$

$\hat{P}\left(\hat{X}_{12} = Z, \hat{X}_{21} = Z\right) = a^2 \geq 1/4$

The event $E = \left\{\hat{X}_{12} = X, \hat{X}_{21} = Z\right\}$ will be *p*-evident for $\sigma = \{1,2\}$ and for some *p* between 0 and 1, if and only if: $P\left(\hat{X}_{12} = Z, \hat{X}_{21} = Z \mid X_1 = X_2 = X\right) \geq p$, that is, if: $a^2 \geq p$. But if $a^2 < \dfrac{U(Z)}{U(\hat{M})}$ then the strategy "attack" will never be a *p*-dominant strategy for any agent. In this case, the ruler could assign the lower endowment to both agents and fear no revolts.

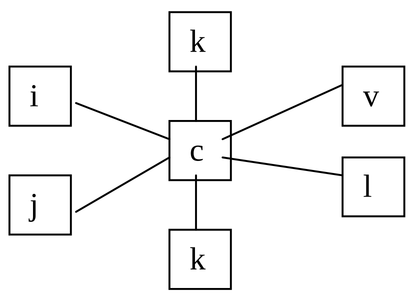
On the other hand, if $a^2 \geq \dfrac{U(Z)}{U(\hat{M})}$, hence at the event $E = \left\{\hat{X}_{12} = Z, \hat{X}_{21} = Z\right\}$ the strategy "attack" for both agents will be a *p*-dominant equilibrium played with probability 1 on such event. Agent zero will dare to take such risk if:

$\left(1 - \hat{P}\left(E; (\Gamma, N), a, \{X_1 = Z, X_2 = Z\}\right)\right)U\left(3M - 2Z\right) = \left(1 - a^2\right)U\left(3M - 2Z\right) \geq U(2M - Z)$

The extreme example of a ruler willing to bear such risk is, of course, a risk neutral one, for whom the inequality will become: $(1-a^2)(3M-2Z)\geq 2M-Z$, recalling $a^2 \geq 1/2$, we get: 2>3; that means agent zero will never take such risk. Hence, any ruler will assign Z to both agents if and only if $a^2 < \dfrac{U(Z)}{U(\hat{M})}$. Otherwise, he will assign Z to one agent and M to the other.

Example 2

A more interesting example is when we have the network:



In this network the salient feature is the central agent *c*, who have contact with every agent in *N*. The remaining agents, let's call them the periphery, have to rely on the information they receive through *c*. There are several possible distributions. Here follows the most important ones.

Assume $\hat{f}$ =3 and the agents in $F \subset N$ $(\#(F)\geq 3$ and *c* is in *F*) get an assignment of *Z*. Let be $\sigma \subset F$ a group with three elements, again including *c*, whose agents receive a signal $E = \{\hat{X}_{st} = Z; s,t \in \sigma\}$. "Attack" will be a *p*-dominant equilibrium played with probability 1 on this event if this condition holds:

$\hat{P}\left(\hat{X}_{ij} = Z \mid X_i = Z; i, j \in \sigma\right) \geq p \geq \frac{U(Z)}{U(\hat{M})}$ That means in this context $a^6 \geq p \geq \frac{U(Z)}{U(\hat{M})}$. If this is true, the probability the ruler is defeated is high, since it is very likely at least 3 agents whose endowment is *Z* receive a signal like *E*.

Another distribution that may improve the ruler's expected utility is to give *M* to *c* and *Z* to everybody else. This means the endowment of the agent will depend on his position in the network: the well connected get higher endowments. Consider then a group $\tilde{\sigma} \subset F$ of 3 elements. "Attack" will be played with probability 1 in the event $E = \left\{\hat{X}_{st} = Z; s, t \in \tilde{\sigma}\right\}$ if this condition holds:

$\hat{P}\left(\hat{X}_{ij} = Z \mid X_i = Z; i, j \in \tilde{\sigma}\right) \geq p \geq \frac{U(Z)}{U(\hat{M})}$. But, since the signals $\hat{X}_{ij} = Z$ are coming from the periphery, this probability is:

$\hat{P}\left(\hat{X}_{ij} = Z \mid X_i = Z; i, j \in \sigma\right) = \left(a^3 + (1-a)^3\right)^3$. Additionally, whenever *a>1/2* we have $\left(a^3 + (1-a)^3\right)^3 < a^6$. So it is possible we have $a^6 \geq p \geq \frac{U(Z)}{U(\hat{M})} > \left(a^3 + (1-a)^3\right)^3$, which means these coalitions are easier to arise when *c* gets *Z* but impossible if agents in the periphery are getting prospective allies only from the periphery itself. In such case, the ruler will safely assign ***Z*** to everybody but agent c, who will get *M*.

Also note that, given the inequality $a^6 \geq p \geq \frac{U(Z)}{U(\hat{M})} > \left(a^3 + (1-a)^3\right)^3$, if the network is full connected (i.e. the degree of separation between any two agents is 1) the extraction level can not be greater than 2.

IV.-DISCUSSION.

VI.1.- Ethnic fractionalization and rent-seeking governments.

The poor economic performance of almost all sub-Saharan countries has haunted economists for decades (see, for example, Collier and Gunning 1999, and references therein). Easterly and Levine (1997) argued that ethnic conflict, which has troubled those countries especially since their independence from European powers, is a major explanation of such a disappointing performance. Alesina et al (2002) confirm a strong relationship between ethnic and linguistic fractionalization on the one hand, and the poor quality of institutions and low growth on the other hand.

Although such studies, based as they are on cross-sectional analysis, shed considerable light on the issue, it is hard to accept a general one-dimensional, unidirectional, and monotonic relationship between ethno-linguistic fractionalization and economic performance (see Esman 1989 and 2002). Instead, case studies should help us to understand better the relationships between these variables. In this section we apply our model of distribution and noise communication in networks to the Nigeria and Congo/Zaire cases.

First, we take into account that the main income of these countries comes not from production but from rents. This is so essentially because of the rich subsoil of these countries coupled with a poor entrepreneurial environment. Nigeria owns oil, which has produced $280 billon in revenues since the discovery of reserves in the late 1950s (Alesina et al). Meanwhile, Congo is rich in such minerals as cobalt, copper, and diamonds, the exportation of which constitutes most of the national taxable income. Hence, we can think this economies as “distributive” instead of “productive”.

Next, note both countries, according to Alesina et al's index,[12] are among the most ethnically and linguistically diverse in the world. Congo has an ethnic fractionalization index of 0.874 while Nigeria's is 0.85. Germany, by comparison, has 0.16. The majority of the Nigerian population is distributed among 350 ethnic groups that are excluded from political power.[13] Therefore, we could use our model representing each ethnic group as a node in the network. The linkages are defined by communication channels among these groups, which are subject to cultural and linguistic restrictions (i.e. it is not the case that there is a fully connected network). Also, it has been documented (Alesina and La Ferrara, 2002) that the trust level is low among people of diverse racial backgrounds. This lack of trust introduces the noise in the communication among the nodes of our network.

In this scenario, our model will predict that an elite or dictator will take advantage of the lack of linkage among the nodes (ethnic groups) and of the noise present in the communication among the nodes, in order to appropriate a significant share of the country's wealth. Also, we should see the ruler preventing the formation of communication channels among the people, breaking them up whenever possible and increasing the distrust (noise) among the nodes as much as he can. In the case of Congo and its dictator/president from 1965 to 1997, Castells (2000, p.100) says: "Mobutu relied on a very simple system of power. He controlled the only operational unit of the army, the presidential guard, and divided politics, government, and army positions among different ethnic groups. He patronized all of them, but also encouraged their violent confrontation." With respect to wealth appropriation by the ruler, note that Mobutu had in 1993 a personal

[12] This index measures the probability that two persons of that country, chosen randomly, happen to belong to different ethnic groups.
[13] Unless explicitly stated, the data and facts about African countries and specifically about Nigeria and Congo are from Castells (2000).

fortune of $10 billon outside his country. More generally, in Sub-Saharan states there are few wealthy individuals, and those few display high levels of consumption while exporting capital to personal accounts in Europe and the U.S. This wealth represents a significant proportion of each country's capital. Meanwhile, most of the population survives under chronic conditions of poverty.

So, according to our analysis, the ruler in each of these countries has chosen an appropriation/distribution strategy, instead of a production/taxation one. This will have an additional effect: The regime will not care about providing a safe environment for business, enforcing property rights and contracts, or providing other public goods, since taxation is not the source of his income. The evidence in Nigeria and Congo could not be clearer.

IV.2.- Common Knowledge and collective action in noisy networks.

In this model, the lack of common knowledge of the distribution of wealth makes it possible for the ruler to increase his expected utility, making an "unfair" and uneven allocation of income available. The specific extraction level the regime could exercise depends on the whole structure of the network and on the channel capacity. In that sense, these two factors define political equilibrium between the government and citizens.[14] We can say, then, that a well-connected network and good communications channels serve as a counterbalance to government power, precluding abusive behavior on the part of the

[14] Those are the only determinants of the extraction power, since the incentive-distorting effect analyzed by Olson (1996) and Acemoglu (2002) is not considered in this model. Also, note that the lack of collective action against the ruler will be due only to the lack of common knowledge, since the free rider effect is not present in this model.

ruler. Also, they facilitate a more equalitarian distribution of wealth by making excessive or non-justified extractions more difficult to carry out.[15]

In that sense, this model helps us to understand why regimes (or rulers in the broader sense) have taken care of the network that facilitates communication among its citizens or subordinates. For example, Chwe (2002) reports that Hawaiian farmers hired workers who did not all speak the same language. Tilly (1997), talking about the Tudors' effort to build a centralized English state, says that they discouraged the cooperation of their dependents and tenants. In the worst moments of some Latin-American dictatorships, people were not allowed to join in groups over a limited number of persons. Communist regimes took care to systematically preclude their citizens from gaining free access to communication devices such as radio transmitters, photocopiers, etc. This was also the case with the European colonialist method in Africa, where "on the one hand there was the legal state, as a racialized entity, under the control of the Europeans; on the other hand was the customary power of native power structures, as an ethnic/tribal identity. The unity of the former and the fragmentation of the latter were essential mechanisms of control under colonial administrations…." (Castells, 2000, p 106).

Networks have been studied in several sciences. Strogatz (2001) and Newman (2002) explain that networks (from neural networks to food webs to semantic linkages) present several statistical similarities, among them "skewed degree distributions." The

[15] An anecdote from Chwe (2001) serves to illustrate how our model works. Chwe relates that in 1977, the Egyptian government announced an increase in the price of bread after 30 years of a frozen price, which provoked major riots and protests against the government. Eventually, the announced increment was rescinded, but loaves of bread were made smaller and were of lower quality. Although everybody noticed the change, it did not count as common knowledge since the government did not announce it. There were no disturbances.

degree of a node is the number of other nodes to which it is connected, and there are usually a small positive number of nodes exhibiting a very high degree.

This knowledge about networks could be useful in understanding political and economic issues using models such as the present one. For example, Barabasy (2002) explains how a network is immune to a relatively short number of random attacks. Since the degree distribution is skewed, however, attacks directed against hubs could seriously affect network connectivity. In our model, this is not hard to analyze if we give the ruler the opportunity to "shape" the network before the citizens' move begins. A repressive regime could then try to eliminate people who are highly connected and, by thus reducing network connectivity, it would be able to increase its expected utility. There is, however, an alternative to treating the well connected: cooptation. If we allow each agent be able to "disconnect" his links (allow no messages to go through his links), an appropriate payment (anything more than or equal to M) to these hubs could be enough to decrease network connectivity significantly.

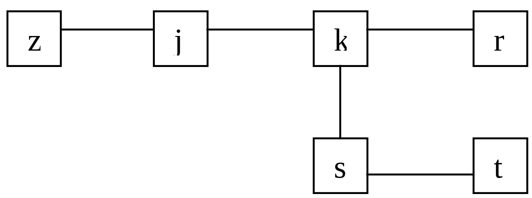
Another implication of this model is the emergence of economic inequality, not only between the ruler and the citizens, but also among the citizens, since the well connected will be more likely to receive a bigger allocation.

"The best common knowledge generator in the U.S. today is network television," says Chwe (1998) in analyzing the role of the media in collective action. In fact, when a citizen learns news from TV, he not only knows it, but also knows it is common knowledge for a great number of people watching the same show. In our model, free media reporting on the distribution of wealth will make the emergence of a successful rent-seeking regime impossible. However, if the government controls the media, the citizens have to rely on their own network to learn about others' situations, which makes wealth extraction possible.

There is an emerging body of literature on the relationship between media and government (Besley and Prat, 2001, Djankov, S. et al, 2002). Although much of this work applies to electoral systems, our model's implications are somewhat consonant with their results, showing that government ownership of media undermines political and economic freedom.

Appendix 1.

First note $P\left(\left\{\hat{X}_{ij}\right\}_{i=1..n}^{j=1..n}\right)=\prod_{j=1}^{n} P\left(\left\{\hat{X}_{ij}\right\}_{i=1..n}\right)$, since the signals agents *1,2,.., s -1, s +1,.., n* receive about X_s are independent of the signals agents *1,2,.., j -1, j +1,.., n* receive about $X_j, s \neq j$. Thinking about $P\left(\left\{\hat{X}_{ij}\right\}_{i=1..n}\right)$, it's clear there could be stochastic dependence among these signals, depending on (Γ, N), that is, depending on the pathway the signals have traveled. Take for example the network:



The probability of this realization of the signals coming from *j* [16]:

$\left\{\hat{X}_{zj}, \hat{X}_{kj}, \hat{X}_{sj}, \hat{X}_{rj}, \hat{X}_{tj}\right\}$ given $X_j = q_l$,is:

$$P\left(\hat{X}_{zj}, \hat{X}_{kj}, \hat{X}_{sj}, \hat{X}_{rj}, \hat{X}_{tj} \mid X_j\right)= P\left(\hat{X}_{zj} \mid X_j\right) P\left(\hat{X}_{kj}, \hat{X}_{rj} \mid X_j\right) P\left(\hat{X}_{sj}, \hat{X}_{tj} \mid \hat{X}_{kj}\right)=$$

$$P\left(\hat{X}_{zj} \mid X_j\right) P\left(\hat{X}_{kj} \mid X_j\right) P\left(\hat{X}_{rj} \mid \hat{X}_{kj}\right) P\left(\hat{X}_{sj} \mid \hat{X}_{kj}\right) P\left(\hat{X}_{tj} \mid \hat{X}_{sj}\right)= P_{zj,j}{}^{(1)} P_{kj,j}{}^{(1)} P_{rj,kj}{}^{(1)} P_{sj,kj}{}^{(1)} P_{tj,sj}{}^{(1)}$$

Generalizing, we could classify the agents depending on its position respect to *j*:

[16] We are noting the event $\left\{\hat{X}_{zj} = q_{zj}, \hat{X}_{kj} = q_{kj}, \hat{X}_{sj} = q_{sj}, \hat{X}_{rj} = q_{rj}, \hat{X}_{tj} = q_{tj}\right\}$ where $q_{ab} \in \{q_1, q_2\}$ as just $\left\{\hat{X}_{zj}, \hat{X}_{kj}, \hat{X}_{sj}, \hat{X}_{rj}, \hat{X}_{tj}\right\}$.

A_1 =The set of “terminal” agents. From these agents no agent receives information about X_j.

$A_2 = \{ s \in N / \exists k \in A_1 : k \ gets \quad \hat{X}_{kj} \ from \quad s \}$

:

:

$$A_m = \{ s \in N / \exists k \in A_{m-1} : k \ gets \quad \hat{X}_{kj} \ from \quad s \}$$

Obviously $m < n$ and $\{A_l, l = 1,2,..n\}$ is a partition of $N \setminus \{j\}$. Note the geodesic going from *i* to *j*:

$$G_{ij} = \{k_1 k_2, k_2 k_3, .., k_{h+1} k_h : k_l k_{l+1} \in \Gamma; l = 1,.., h; k_1 = i; k_m = j\}$$

And:

$$G_j = \bigcup_{i=1}^{n} G_{ij}$$

Hence, G_i is the collection of all the links that conform the geodesics going from *j* to every other agent in N.

Using this notation and having in mind that signals travel across the network as Markov chains, we can finally write:

$$P\left(\left\{\hat{X}_{ij}\right\}_{i=1..n}\right) = \prod_{l-1}^{m} \prod_{a(l) \in A_i} P\left(\hat{X}_{a(l)j} \mid \hat{X}_{bj}, b \in A_{l+1}\right) = \prod_{l-1}^{m} \prod_{a(l) \in A_i} P\left(\hat{X}_{a(l)j} \mid \hat{X}_{bj}, b \in A_{l+1}, a(l)b \in G_i\right)$$

We’re noting $\{\hat{X}_{bj} = X_j, b \in A_{m+1}\}$ for $\{\hat{X}_{bj}, b \in A_{m+1}\}$. This completes the explicit definition of *P(.)*.

Appendix 2.

Lemma 1. The necessary and sufficient condition for $E = \{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in \sigma\}$ being common *p*-believed in *E* (i.e. $C^p(E) = E$) is $\hat{P}(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma) \geq p$.

The first step is to find $B_i^p(E)$ for any $i \in \sigma$, which is $B_i^p(E) = \{\omega \in \Omega / P_i(E \mid \Psi_i(\omega)) \geq p\}$. The only candidates to be elements of $B_i^p(E)$ are the ω such that $\Psi_i(\omega) = \{\omega \in \Omega / \hat{X}_{ik} = Z, \forall k \in \sigma\}$. Since the probability i assigns to *E* only depends on the information i receives, we have just two possibilities: $B_i^p(E) = \Psi_i(\omega)$ or $B_i^p(E) = \phi$. This is true because any two elements in a set of i's partition yields the same signals to i. Hence we can say:

$$B_i^p(E) = \{\omega \in \Omega / \hat{X}_{ik} = Z; k \in \sigma\} \Leftrightarrow P_i(X_k = Z, \hat{X}_{tk} = Z \mid \hat{X}_{ik} = Z; k, t \in \sigma, i \neq t) \geq p$$

and $B_i^p(E) = \phi \Leftrightarrow P_i(X_k = Z, \hat{X}_{tk} = Z \mid \hat{X}_{ik} = Z; k, t \in \sigma, i \neq t) < p$

So, the task is now to calculate the conditional probability in the left side of this bi-conditional. Again, it will depend on the network structure and noise level:

$$P_i(X_k = Z, \hat{X}_{tk} = Z \mid \hat{X}_{ik} = Z; k, t \in \sigma, t \neq i) = \frac{P_i(X_k = Z, \hat{X}_{tk} = Z; k, t \in \sigma)}{P_i(\hat{X}_{ik} = Z; k \in \sigma)}$$

The expression in the denominator is not hard to calculate: given our Laplacian priors and the stochastic independence of the signals originated in different agents, the probability that agent i receives *f-1* signals *Z* is $(1/2)^{f-1}$. The numerator is calculated as follows:

$P_i\left(X_k = Z, \hat{X}_{tk} = Z; k, t \in \sigma\right) = \hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right) P_i\left(X_k = Z; k \in \sigma, k \neq i\right)$ Again, the priors we've assumed tell us: $P_i\left(X_k = Z; k \in \sigma, k \neq i\right) = (1/2)^{f-1}$. On the other hand, $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right)$ -which is the probability everybody in σ receives and Z from everybody in σ, given that everybody in σ has an endowment of Z- depends on the network structure (Γ, N) and on the noise level $1-a$. We have dropped the sub index in that expression, because it is equal for every agent in σ [17], and added a hat to P because it is the same probability distribution we explain in Appendix 1. The precise way to calculate such value also could be seen in the examples above, for now let's write:

$$P_i\left(X_k = Z, \hat{X}_{tk} = Z \mid \hat{X}_{ik} = Z; k, t \in \sigma, t \neq i\right) = \hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right)$$

So, if and only if the condition $\hat{P}\left(\hat{X}_{tk} = Z \mid X_k = Z; k, t \in \sigma\right) \geq p$ holds, we can say $B_i^p(E) = \Psi_i(E) = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k \in \sigma\right\}$. The reasoning is the same for all agents in σ, hence it is the condition necessary and sufficient to say $B^p(E) = \bigcap_{i \in \sigma} B_i^p(E) = \bigcap_{i \in \sigma} \Psi_i(\omega) = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in \sigma\right\} = E$ and following to higher order beliefs: $B^p \ldots B^p(E) = \left\{\omega \in \Omega / \hat{X}_{ik} = Z; k, i \in \sigma\right\} = E$. That completes the proof.

[17] Note every agent is getting just "Z" as signals from σ.

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